post mortem, this, which has always been denied by Virchow and others, is negatived by the occurrence in the stools here examined of an abundance of epithelial cells, often very slightly differing in appearance from the normal. Occasionally they are coherent as groups of four and five; there are, however, no finger-shaped casts of complete villi.

The cases of a doubtful nature from Venetia have not disclosed any comma-bacilli under microscopical examination. In one of them the ulcers present in the ileum, which to the naked eye resembled those of enteric fever, pass deeply into the thickness of the muscular coat of the intestine, a condition to which I have only once seen any close approach in Asiatic cholera.

XXXII. "On certain Definite Integrals. No. 15." By W. H. L. Russell, F.R.S. Received June 16, 1887.

Mr. Fox Talbot's researches on the comparison of transcendents are well known. The following are founded on the same principle, applied in a different manner:—

Let—

$$a_1x^3+b_1y^3+c_1z^3 = e_1,$$

$$a_2x^3+b_2y^3+c_2z^3 = e_2,$$

be two equations connecting the variables x, y, and z. Then we can find x and y in terms of z, x and z in terms of y, y and z in terms of (x). Or if—

$$b_1c_2-c_1b_2 = A_1, \qquad c_1a_2-a_1c_2 = B_1, \qquad a_1b_2-b_1a_2 = C_1;$$

and also

$$e_1a_2-e_2a_1 = A_2, \qquad e_1b_2-e_2b_1 = B_2, \qquad e_1c_2-e_2c_1 = C_2;$$

we shall have
$$x = \frac{1}{\sqrt[3]{(C_1)}}\sqrt[3]{(B_2+A_1z^3)},$$

$$y = -\frac{1}{\sqrt[3]{(C_1)}}\sqrt[3]{(A_2-B_1z^3)},$$

$$z = \frac{1}{\sqrt[3]{(B_1)}}\sqrt[3]{(A_2+C_1y^3)},$$

$$x = -\frac{1}{\sqrt[3]{(B_1)}}\sqrt[3]{(C_2-A_1y^3)},$$

$$y = \frac{1}{\sqrt[3]{(A_1)}}\sqrt[3]{(C_2+B_1x^3)},$$

$$z = -\frac{1}{\sqrt[3]{(A_1)}}\sqrt[3]{(B_2-C_1x^3)}.$$

Now, since

$$\int dx\,.\,zy+\int dy\;xz+\int dz\;xy = xyz,$$

$$(B_1C_1)^{\frac{1}{3}}\int_{\lambda_2}^{\lambda_1} dx\,\sqrt[3]{(C_2+B_1x^3)}\,\sqrt[3]{(B_2-C_1x^3)}$$

$$+(A_1C_1)^{\frac{1}{3}}\int_{\mu_2}^{\mu_1} dx\,\sqrt[3]{(A_2+C_1x^3)}\,\sqrt[3]{(C_2-Ax^3)}$$

$$+(A_1B_1)^{\frac{1}{3}}\int_{\nu_2}^{\nu_1} dx\,\sqrt[3]{(B_2+A_1x^3)}\,\sqrt[3]{(A_2-B_1x^3)}$$

$$= (A_1B_1C_1)^{\frac{1}{3}}(\lambda_2\mu_2\nu_2-\lambda_1\mu_1\nu_1),$$

where the limits $\lambda_1\lambda_2\mu_1\mu_2\nu_1\nu_2$ must satisfy the equations—

$$a_1\lambda_1^3+b_1\mu_1^3+c_1\nu_1^3 = e_1,$$

$$a_1\lambda_2^3+b_1\mu_2^3+c_2\nu_2^3 = e_2;$$

$$a_2\lambda_1^3+b_2\mu_1^3+c_2\nu_2^3 = e_2,$$

$$a_2\lambda_2^3+b_2\mu_2^3+c_2\nu_2^3 = e_2;$$

from which it appears that one pair of limits may be considered arbitrary, while the other two pairs are given by these equations.

Since—

$$\int\frac{dx}{x^2yz}+\int\frac{dy}{xy^2z}+\int\frac{dz}{xyz^2} = -\frac{1}{xyz},$$

$$A_1^{\frac{2}{3}}\int_{\lambda_2}^{\lambda_1}\frac{dx}{x^3\sqrt[3]{(C_2+B_1x^3)}\,\sqrt[3]{(B_2-C_1x^3)}}$$

$$+B_1^{\frac{2}{3}}\int_{\mu_2}^{\mu_1}\frac{dx}{x^2\sqrt[3]{(A_2+C_1x^3)}\,\sqrt[3]{(C_2-A_1x^3)}}$$

$$+C_1^{\frac{2}{3}}\int_{\nu_2}^{\nu_1}\frac{dx}{x^2\sqrt[3]{(B_2+A_1x^3)}\,\sqrt[3]{(A_2-B_1x^3)}}$$

$$= \lambda_1\mu_1\nu_1-\lambda_2\mu_2\nu_2,$$

the limits being determined as before; and since—

$$\int\frac{dx}{yz}-\int\frac{xdy}{y^2z}-\int\frac{xdz}{z^2y} = \frac{x}{yz},$$

$$\int_{\mu_2}^{\mu_1}\frac{dx}{x^2}\,\frac{\sqrt[3]{(C_2-A_1x^3)}}{\sqrt[3]{(A_2+C_1x^3)}}+\int_{\nu_2}^{\nu_1}\frac{dx}{x^2}\,\frac{\sqrt[3]{(B_2+A_1x^3)}}{\sqrt[3]{(A_2-B_1x^3)}}$$

$$-A_1^{\frac{2}{3}}\int_{\lambda_2}^{\lambda_1}\frac{dx}{\sqrt[3]{(C_2+B_1x^3)}\,\sqrt[3]{(B_2-C_1x^3)}} = \frac{\lambda_1}{\mu_1\nu_1}-\frac{\lambda_2}{\mu_2\nu_2};$$

moreover, since

$$\int\frac{z}{x}\,dy+\int\frac{y}{x}\,dz-\int\frac{yz}{x^2}\,dx = \frac{yz}{x},$$

we shall have $\displaystyle\int_{\mu_2}^{\mu_1}dx\,\frac{\sqrt[3]{(A_2+C_1x^3)}}{\sqrt[3]{(C_2-A_1x^3)}}+\int_{\nu_2}^{\nu_1}dx\,\frac{\sqrt[3]{(A_2-B_1x^3)}}{\sqrt[3]{(B_2+A_1x^3)}}$

$$-\frac{1}{A^{\frac{2}{3}}}\int_{\lambda_2}^{\lambda_1}\frac{\sqrt[3]{(C_2+B_1x^3)}\,\sqrt[3]{(B_2-C_1x^3)}}{x^2} = \frac{\lambda_2}{\mu_2\nu_2}-\frac{\lambda_1}{\mu_1\nu_1};$$

Again, since

$$\int dx\;y^5z^2+5\int dy\;y^4z^2x+2\int z\;dz\;xy^5 = xy^5z^2,$$

$$2A_1^2B_1\sqrt[3]{(A)_1}\int x\;dx\;(B_2+A_1x^3)^{\frac{1}{3}}(A_2-B_1x^3)^{\frac{5}{3}}$$

$$+5A_1^2C_1^2\sqrt[3]{(A_1)}\int dx\;x^4(A_2+C_2x^3)^{\frac{2}{3}}(C_2-A_1x^3)^{\frac{1}{3}}$$

$$-B_1C_1^2\int dx\;(C_2+B_1x^3)^{\frac{5}{3}}(B_2-C_1x^3)^{\frac{2}{3}}$$

$$= A_1^2B_1C_1^2\sqrt[3]{(A_1)}\,(\lambda_2\mu_2^5\nu_2^3-\lambda_1\mu_1^5\nu_1^3).$$

Similarly, since

$$\int(y+z)\;dx+\int\;(x+z)\;dy+\int\;(x+y)dz = xy+xz+yz,$$

$$\sqrt[3]{(B_1C_1)}\int\;dx\;\{\sqrt[3]{(C_2+B_1x^3)}-\sqrt[3]{(B_2-C_1x^3)}\}$$

$$+\sqrt[3]{(A_1C_1)}\int\;dx\;\{\sqrt[3]{(A_2+C_1x^3)}-\sqrt[3]{(C_2-A_1x^3)}\}$$

$$+\sqrt[3]{(A_1B_1)}\int\;dx\;\{\sqrt[3]{(B_2+A_1x^3)}-\sqrt[3]{(A_2-B_1x^3)}\}$$

$$= \sqrt[3]{(A_1B_1C_1)}\{\Sigma\lambda_2\mu_2-\Sigma\lambda_1\mu_1\}.$$

Next suppose that we transform by means of the equations—

$$a_1x+b_1y+c_1z = e_1,$$

$$a_2x+b_2y+c_2z = e_2,$$

and remember that ϕ_1, ϕ_2, ϕ_3 being any functions—

$$\int dx\,\phi_1'(x)\,\phi_2(y)\,\phi_3z+\int dy\,\phi_2'y\,\phi_1(x)\,\phi_3(z)$$

$$+\int dz\,\phi_3'(z)\,\phi_1(x)\,\phi_2(y) = \phi_1x\,\phi_2y\,\phi_3z;$$

then, transforming as before (with similar limits),

$$\int dx\,\phi_1'(x)\,\phi_2\frac{B_1x+C_2}{A_1}\,\phi_3\frac{C_1x-B_2}{A_1}$$

$$+\int dx\,\phi_2'(x)\,\phi_3\frac{C_1x+A_2}{B_1}\,\phi_1\frac{A_1x-C_2}{B_1}$$

$$+\int dx\,\phi_3'(x)\,\phi_1\frac{A_1z+B_2}{C_1}\,\phi_2\frac{B_1x-A_2}{C_2}$$

$$= \phi_1\lambda_1\,\phi_2\mu_2\,\phi_2\nu_2-\phi\lambda_2\,\phi_2\mu_2\,\phi_3\nu_2,$$

we observe there are three arbitrary functions, which can be taken at pleasure, and six arbitrary constants. We perceive therefore that the formula is very extensive.

The limits are of course connected by the equations—

$$a_1\lambda_1+b_1\mu_1+c_1\nu_1 = e_1,$$

$$a_1\lambda_2+b_1\mu_2+c_1\nu_2 = e_1,$$

$$a_2\lambda_1+b_2\mu_1+c_2\nu_1 = e_2,$$

$$a_2\lambda_2+b_2\mu_2+c_2\nu_2 = e_2.$$

Next let us transform by means of the equations—

$$a_1(x^2-2mx) + b_1(y^2-2my) + c_1(z^2-2\nu z) = e_1,$$

$$a_2(x^2-2mx) + b_2(y^2-2my) + c_2(z^2-2\nu z) = e_2;$$

then, proceeding as before,—

$$x = m + \left\{ m^2 + \frac{B_2}{C_1} - \frac{2rA_1}{C_1} z + \frac{A_1 z^2}{C_1} \right\}^{\frac{1}{2}},$$

$$y = n + \left\{ n^2 - \frac{A_2}{C_1} - \frac{2rB_1}{C_1} z + \frac{B_1 z^2}{C^1} \right\}^{\frac{1}{2}},$$

$$z = r + \left\{ r^2 + \frac{A_2}{B_1} - \frac{2nC_1{}^2}{B_1} y + \frac{C_1 y^2}{A_1} \right\}^{\frac{1}{2}},$$

$$x = m + \left\{ m^2 - \frac{C_2}{B_1} - \frac{2nA}{B_1} y + \frac{A_1 y^2}{B_1} \right\}^{\frac{1}{2}},$$

$$y = n + \left\{ n^2 + \frac{C_2}{A_1} - \frac{2mB_1}{A_1} x + \frac{B_1}{A_1} x^2 \right\}^{\frac{1}{2}},$$

$$z = r + \left\{ r^2 - \frac{B_2}{A_1} - \frac{2mC_1}{A_1} x + \frac{C_1}{A_1} x^2 \right\}^{\frac{1}{2}}.$$

Hence we have the sum of the integrals (supposed to be taken within the proper limits)—

$$\int dx \left\{ m^2 + \frac{B_2}{C_1} - \frac{2rA_1}{C_1} x + \frac{A_1 x^2}{C'} \right\}^{\frac{1}{2}}$$

$$\left\{ n^2 + \frac{A_2}{C_1} - \frac{2rB_1}{C'} x + \frac{B_1 x^2}{C_1} \right\}^{\frac{1}{2}}$$

$$+ \int dx \left\{ r^2 + \frac{A_2}{B_1} - \frac{2nC_1}{B_1} x + \frac{C_1 x^2}{B_1} \right\}^{\frac{1}{2}}$$

$$\left\{ m^2 - \frac{C_2}{B_1} - \frac{2nA_1}{B_1} x + \frac{A_1 x^2}{B_1} \right\}^{\frac{1}{2}}$$

$$+ \int dx \left\{ n^2 + \frac{C_2}{A_1} - \frac{2mB_1}{A_1} x + \frac{B_2}{A_1} x^2 \right\}^{\frac{1}{2}}$$

$$\left\{ r^2 - \frac{B_2}{A_1} - \frac{2mC_1}{A_1} + \frac{C_1 x^2}{A_1} \right\}^{\frac{1}{2}},$$

expressed by elliptic functions.

As a final example take the following:—

Let—
$$a_1 x^3 + b_1 y^5 + c_1 z^3 = e_1,$$
$$a_2 x^3 + b_2 y^5 + c_2 z^3 = e_2.$$

Then since—

$$\int dx\, yz + \int dy\, xz + \int dz\, xy = xyz,$$

$$\sqrt[15]{(B_1^{10}C_1^8)}\int dx\ \sqrt[5]{(C_2+B_1x^3)}\sqrt[3]{(B_2-C_1x^3)}$$

$$+\sqrt[15]{(A_1^8C_1^8)}\int dx\ \sqrt[3]{(A_2+C_1x^5)}\sqrt[3]{(C_2-A_1x^5)}$$

$$+\sqrt[15]{(B_1^{10}A_1^8)}\int dx\ \sqrt[3]{(B_2+A_1x^3)}\sqrt[5]{(A_2-B_1x^3)}$$

$$=\sqrt[15]{(A^8B^{10}C_1^8)}(\lambda_2\mu_2\nu_2-\lambda_1\mu_1\nu_1).$$

If we put

$$\int dx\ yzw+\int dy\ xzw+\int dz\ xy\ w+\int dw\ .\ xyz\ =\ xyzw,$$

and transformed by three equations similar *mutatis mutandis* to those we have have used, we should of course obtain the sum of four definite integrals.

The limits are omitted in some of these equations, but they will be easily seen from the foregoing.

XXXIII. "A Geometrical Interpretation of the first two Periods of Chemical Elements following Hydrogen, showing the Relations of the fourteen Elements to each other and to Hydrogen by means of a Right Line and Cubic Curve with one real Asymptote." By Rev. SAMUEL HAUGHTON, M.D., F.R.S. Received April 30, 1887.

[Publication deferred.]

XXXIV. "On the Force with which the two Layers of the healthy Pleura cohere." By SAMUEL WEST, M.D., F.R.C.P. Communicated by Sir JAMES PAGET, Bart., F.R.S. Received May 21, 1887.

[Publication deferred.]

XXXV. "Total Eclipse of the Sun observed at the Caroline Islands on May 6, 1883." By W. DE W. ABNEY, Capt. R.E., F.R.S. Received May 25, 1887.

[Publication deferred.]

XXXVI. "Note on Mr. Davison's Paper on the Straining of the Earth's Crust in Cooling." By G. H. DARWIN, M.A., F.R.S., Plumian Professor of Astronomy and Experimental Philosophy in the University of Cambridge. Received June 15, 1887.

[To be published in the 'Philosophical Transactions,' in conjunction with Mr. Davison's paper.]

XXXVII. "A further minute Analysis, by Electric Stimulation, of the so-called Motor Region of the Cortex Cerebri in the Monkey (*Macacus sinicus*)." By CHARLES E. BEEVOR, M.D., and Professor VICTOR HORSLEY, F.R.S., B.S., F.R.C.S. Abstract received June 16, 1887.

[Publication deferred.]

XXXVIII. "On the present Position of the Question of the Sources of the Nitrogen of Vegetation, with some new Results, and preliminary Notice of new Lines of Investigation." By Sir J. B. LAWES, Bart., F.R.S., and J. H. GILBERT, M.A., LL.D., F.R.S., Sibthorpian Professor of Rural Economy in the University of Oxford. Abstract received June 16, 1887.

[Publication deferred.]

XXXIX. "On Diameters of Plane Cubics." By JOHN J. WALKER, M.A., F.R.S. Received June 16, 1887.

[Publication deferred.]

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